An Introduction to the Single Slit Diffraction

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1 Introduction

The single-slit experiment is one of the most famous experiments known in classical physics. Its core result is the observation of the wave nature of light.

Let us set up the experiment. We place a monochromatic light source some distance away from a solid barrier with a narrow slit of width a. We assume the source is very far away to make the approximation that the light waves are coming in parallel to the barrier. We then place a light detector, which measures the intensity of the light hitting each location, parallel to the barrier some distance L away. For a visual, see Figure 1.

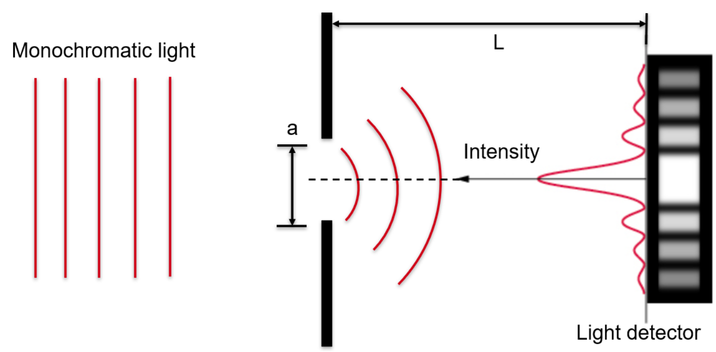

Figure 1: Pictorial depiction of the single slit experiment

If one had to make an educated guess as to the profile of the light intensity on the light detector, one might guess one large maximum intensity centered on the axis down the middle of the slit, and a monotonic symmetric decrease away from the center axis. However, this is not what we observe! In Figure 2, we see the intensity profile for red light for three different slit widths. The profile has one large intensity in the middle, but there are oscillations between no light and smaller submaxima of smaller and smaller intensity.

Our goal now is to determine why the light intensity follows this pattern. To do this, we introduce two concepts that will be used in the mathematical analysis: Huygens' Principle and wave interference.

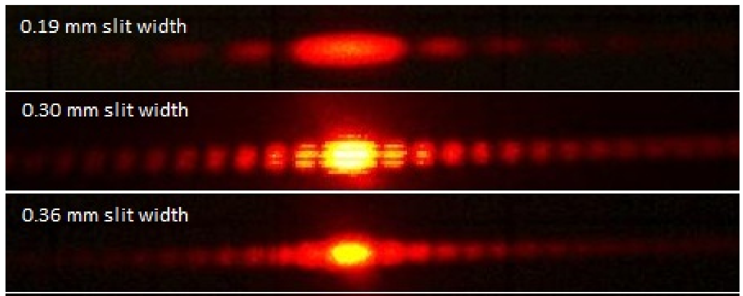

Figure 2: Intensity results for red light for varying slit widths. Note how the minima spacing becomes further apart with smaller slit width.

2 Wave Interference

The superposition of waves is known as interference. The two main types of interferences are known as constructive and destructive interference. Constructive interference occurs when waves are in phase, while destructive interference occurs when the waves are out of phase, with largest effect when they are 90 degrees out of phase.

3 Huygens' Principle

Dutch Scientist Christiaan Huygens proposed a theory of light propagation that helps to establish why light waves bend around a slit. He proposed that a wavefront, which is a crest or trough of light, is composed of wavelets. Each wavelet on the wavefront acts as its own source of light that spreads out in the direction the light is moving. When there are no barriers, interferences between waves leads to one large wavefront. However, when light enters the slit, the

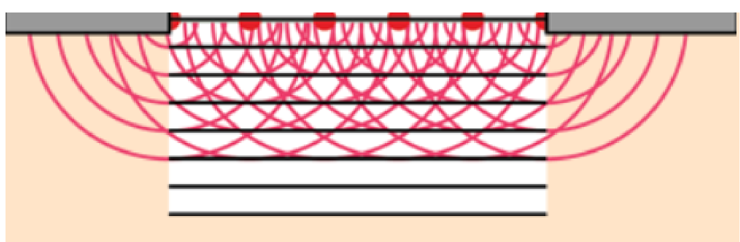

Figure 3: Pictorial depiction of Huygens' Principle

wavelets toward the edge have portions of the light which are not canceled out, which gives the circular wavefront effect observed in experiment.

4 Derivation of the Wave Intensity

To derive the intensity of the light that appears on the screen, we approach the problem by looking at the resultant amplitude contributed by a finite number of equally spaced wavelets between the slit and taking the limit as the number of wavelets goes to infinity. Another way think of this is by splitting the wide slit into the limit of a number of very thin slits.

Suppose we divide the slit into N equally spaced wavelets, as described by Huygens' Principle. Let the distance between each wavelet be $\Delta y = a/N$ apart. We make the following assumption: Assume that the distance to the screen L is much larger than the width of the slit a. With this assumption, we can thus further assume that all the beams of light hit the location located θ degrees above (or below) the center axis are all parallel. Lastly, each wavelet beam will have an amplitude of $\Delta E_0 = \frac{E_0}{N}$ where E_0 is the amplitude of the original wave front entering the slit. This is visualized in Figure 4 below. With the parallel line

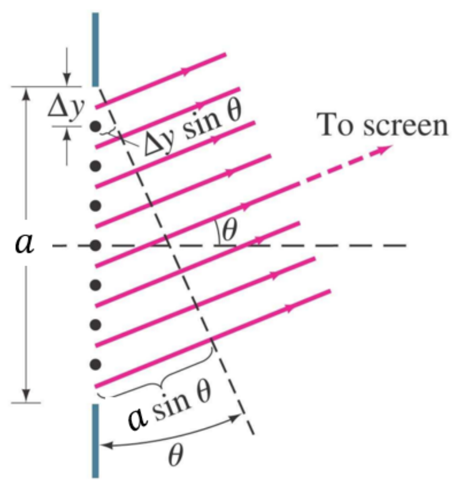

Figure 4: Analytical setup to determine the intensity

assumption, we can set up a right triangle and use simple trig to determine that the path difference between the topmost wavelet and the bottommost wavelet is $a \sin\theta$. Similarly, the path difference between the any two adjacent wavelet beams is $\Delta y \sin\theta$. Since we know the path difference length, we can obtain the phase difference $\Delta\beta$, by setting up the simple proportionality that

$$\frac{\Delta\beta}{2\pi} = \frac{\Delta y \sin\theta}{\lambda}$$

Hence, each wavelet beam is a wave of the following form:

$$\Delta E_0 \sin(\omega t + \Delta\beta n)$$

where ω is the frequency of the light. Thus, by using the superposition of the waves, the total amplitude E_θ at the position θ on the screen will be governed by the sum of each wave $\Delta E_0 \sin(\omega t + \Delta\beta n)$ from each of the N wavelets, with the limit as N goes to infinity.

$$\begin{aligned} E_\theta \sin(\omega t + \phi) &= \lim_{N\to\infty} \frac{E_0}{N} \sum_{n=0}^{N-1} [\sin(\omega t + \Delta\beta n)] \\ &= \lim_{N\to\infty} \frac{E_0}{a} \sum_{n=0}^{N-1} \left[\Delta y \sin(\omega t + \frac{2\pi \sin\theta}{\lambda} n\Delta y)\right] \end{aligned}$$

where we used that $1/N = \Delta y/a$ and the definition of $\Delta\beta$ found above. In the limit, we obtain a definite integral.

$$E_\theta \sin(\omega t + \phi) = \frac{E_0}{a} \int_0^a \sin(\omega t + \frac{2\pi \sin\theta}{\lambda} y) dy$$

After taking the integral, we get that

$$\begin{aligned} E_\theta \sin(\omega t + \phi) &= \frac{-\lambda E_0}{2\pi a \sin\theta} \cos(\omega t + \frac{2\pi \sin\theta}{\lambda} y)\Bigg|_0^a \\ &= \frac{-\lambda E_0}{2\pi a \sin\theta} \left[\cos(\omega t + \frac{2\pi \sin\theta}{\lambda} a) - \cos(\omega t)\right] \end{aligned}$$

Now use the identity that $\cos(A) - \cos(B) = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$ with $A = \omega t + \frac{2\pi \sin\theta}{\lambda} a$ and $B = \omega t$. After plugging in terms and simplifying:

$$E_\theta \sin(\omega t + \phi) = \frac{\lambda E_0}{\pi a \sin\theta} \sin(\omega t + \frac{\pi \sin\theta}{\lambda} a) \sin(\frac{\pi \sin\theta}{\lambda} a)$$

Let $\beta = \frac{2\pi \sin\theta}{\lambda} a$, then by plugging into the above:

$$E_\theta \sin(\omega t + \phi) = E_0 \frac{\sin(\beta/2)}{(\beta/2)} \sin(\omega t + \frac{\beta}{2})$$

So, the phase of the wave hitting at position θ away from the center axis is $\phi = \beta/2$ and the new amplitude is $E_\theta = E_0 \frac{\sin(\beta/2)}{(\beta/2)}$. Lastly, we recall note that light intensity I is directly proportional to the amplitude squared of the wave so

$$I(\theta) \propto E_0^2 \frac{\sin^2(\beta/2)}{(\beta/2)^2}$$

This intensity exactly follows the observed phenomenon as shown in Figures 1 and 2.

5 Analysis

Given the formula for the light intensity, we can analyze what happens to the diffraction pattern as we change the parameters of the experiment. Note that as the wavelength increases, β decreases. Mathematically, the intensity curve will stretch out in the input dimension, and so we would see the bright spots spread out. Similarly, if the width of the slit increases, β increases. Thus the intensity curve will compress in the input dimension, and so we would observe the bright spots to come closer together. This is exactly what happens in Figure 2.

6 References

1. David Morin. https://bpb-us-e1.wpmucdn.com/sites.harvard.edu/dist/0/550/files/2023/11/waves_interference.pdf

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