Single Slit Diffraction
From Experiment to Theory

DEPARTMENT OF MATHEMATICAL SCIENCES
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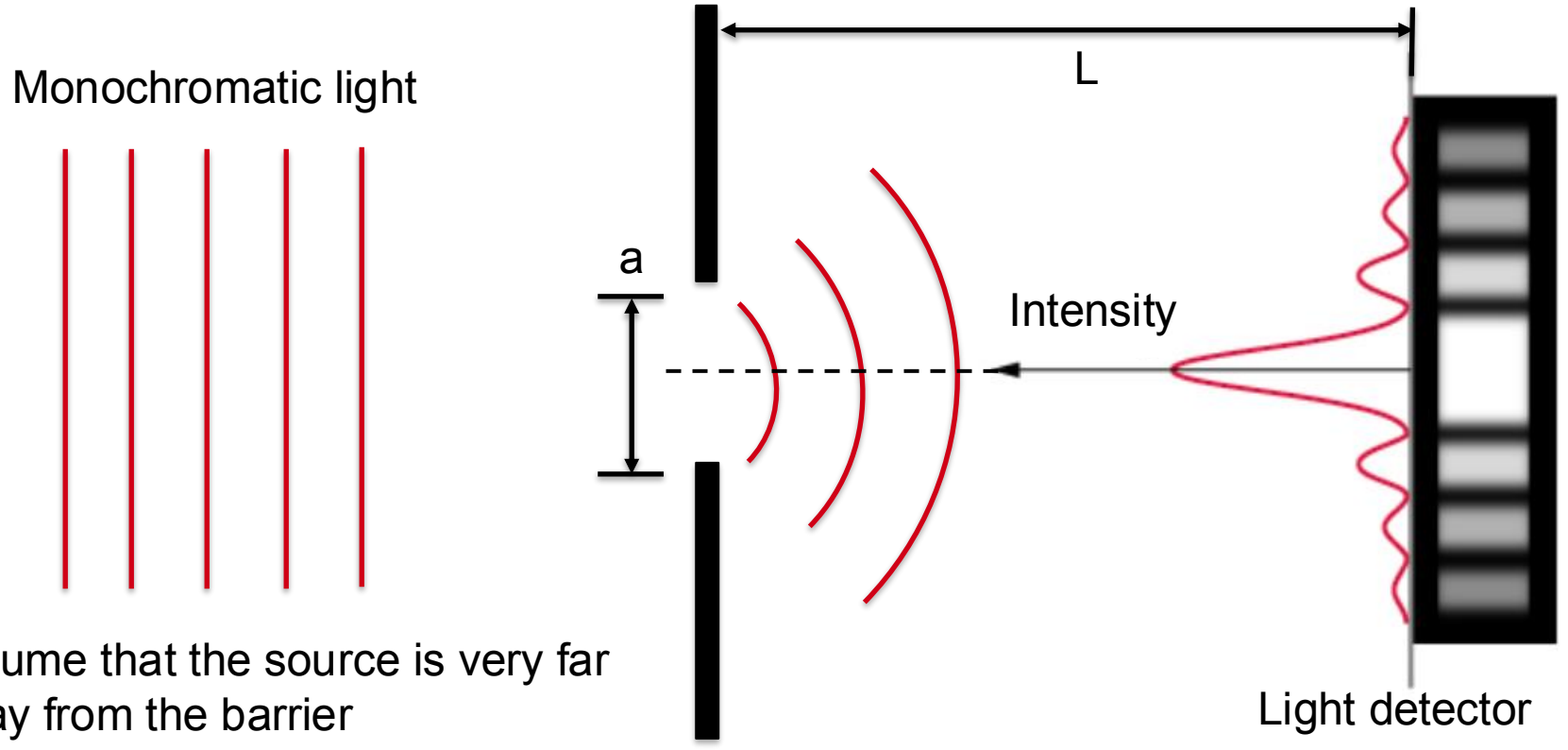
2/25/25

RPI 200

Experimental Setup and Observation



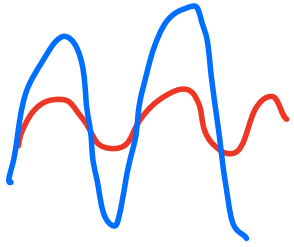
Assume that the source is very far away from the barrier

Questions one might ask

- Why does the diffraction pattern appear?
- How does the hole width (a) and wavelength (λ) change the diffraction pattern?

Interference

- Characteristic of waves caused by the **superposition** of waves
- **Constructive interference**: waves are in phase leading to larger amplitude
- **Destructive interference**: waves are out of phase, with largest effect when they are 90 degrees out of phase, leading to a decrease in amplitude

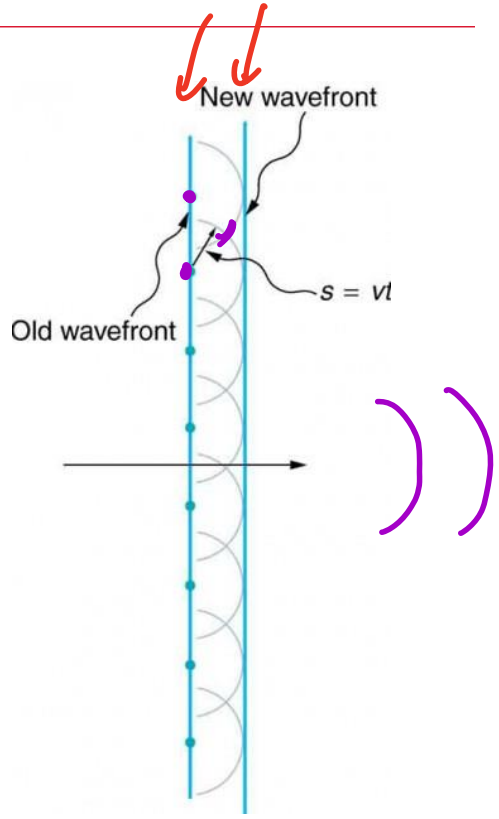


Huygens' Principle

- Due to Dutch Scientist Christiaan Huygens (1629-1695)
- **Wavefront**: a crest or a trough that moves at speed v

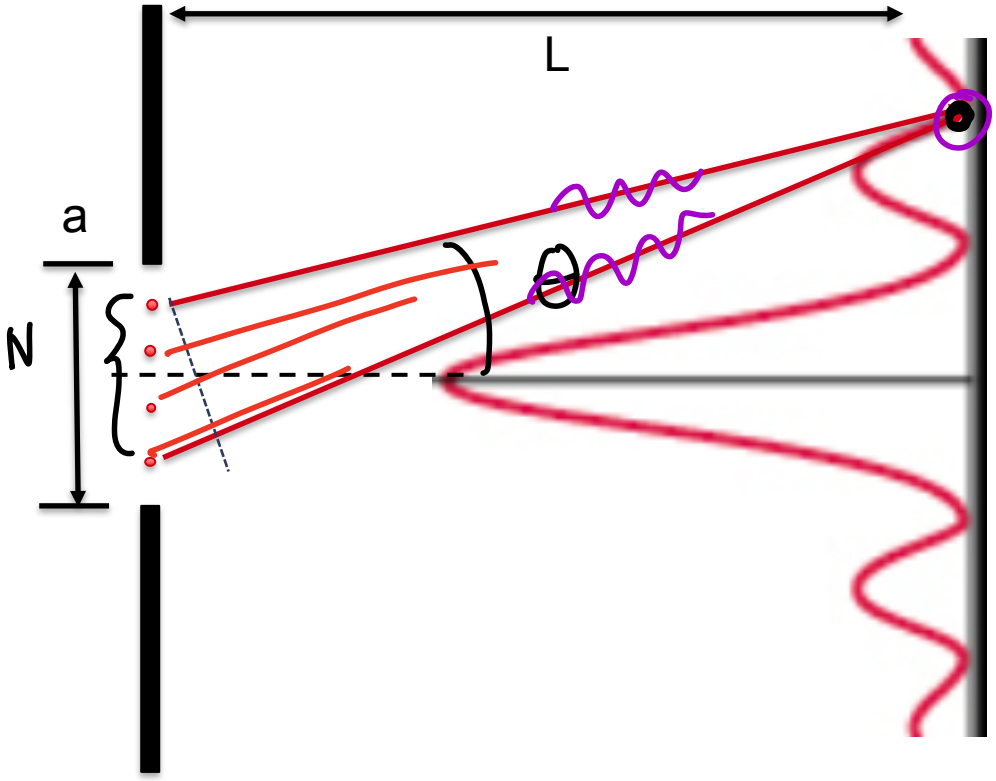
Every point on the wavefront acts as its own source of light, known as a **wavelet**, that spread out in the direction the light travels and at the same speed as the light itself.

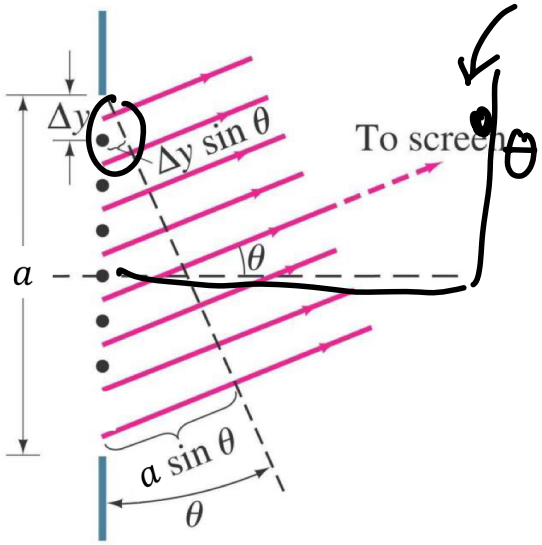
- These wavelets interfere and lead to one large wavefront



General Idea for Derivation of Intensity Curve

- Apply wave interference and Huygens' Principle
- Sketch:
  - Split wavefront into N equally spaced wavelets
  - add up all waves hitting a specific location angle θ away from center axis
  - Take the limit as $N \to \infty$





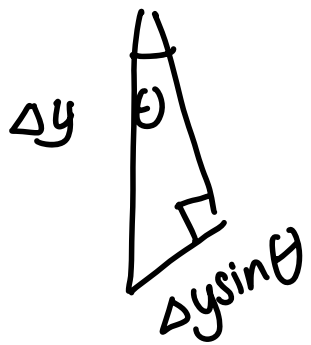
N wavelets

$$\frac{\Delta y}{a} = \frac{1}{N}$$

$$\Delta E_0 \sin(\omega t + \phi_n)$$

$L >> a$, $\Rightarrow$ assume beams are all parallel

$\Delta\beta$ ← phase difference. b/w adjacent wavelets



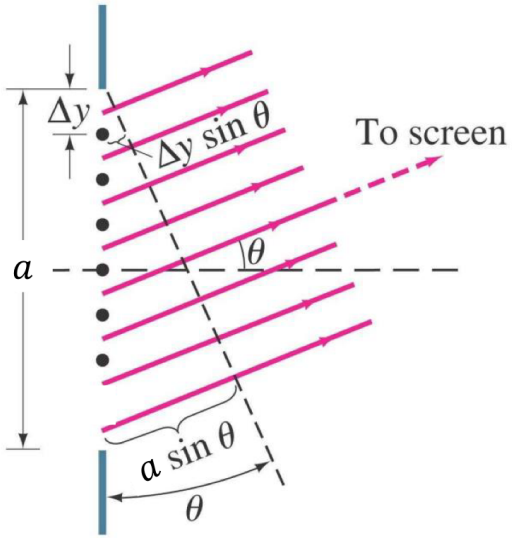
$$\frac{\Delta\beta}{2\pi} = \frac{\Delta y \sin\theta}{\lambda} \Rightarrow \Delta\beta = \frac{2\pi \Delta y \sin\theta}{\lambda}$$

Each wavelet wave looks like:

$$\frac{E_0}{N} \sin(\omega t + \Delta\beta n) \qquad n = 0, 1, 2, \cdots, N-1$$

$$E_0 \sin(\omega t + \phi) = \lim_{N\to\infty} \sum_{n=0}^{N-1} \frac{E_0}{N} \sin(\omega t + \Delta\beta n) \qquad \frac{1}{N} = \frac{\Delta y}{a}$$

$$= \lim_{N\to\infty} \frac{E_0}{a} \sum_{n=0}^{N-1} \left[\Delta y \sin\left(\omega t + \frac{2\pi \sin\theta}{\lambda} n \Delta y\right) \right]$$



$$= \lim_{N\to\infty} \frac{E_0}{a} \sum_{n=0}^{N-1} \left[\Delta y \sin\left(\omega t + \frac{2\pi \sin\theta}{\lambda} \underbrace{n\Delta y}_{y}\right)\right]$$

$$= \frac{E_0}{a} \int_0^a \sin\left(\omega t + \frac{2\pi \sin\theta}{\lambda} y\right) dy$$

$$= \frac{-\lambda E_0}{2\pi a \sin\theta} \cos\left(\omega t + \frac{2\pi \sin\theta}{\lambda} y\right)\Big|_0^a$$

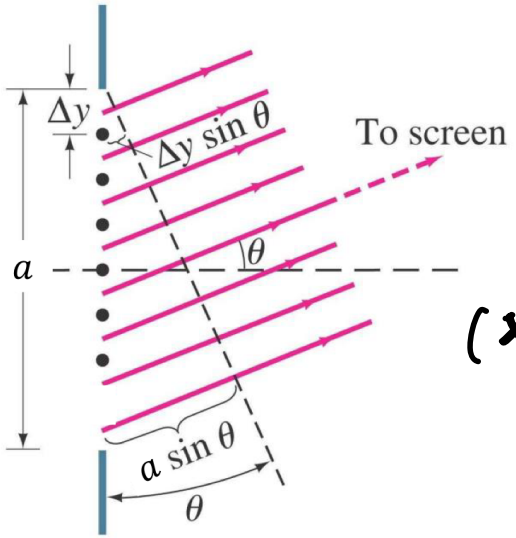
$$(*) = \frac{-\lambda E_0}{2\pi a \sin\theta} \left[\cos\left(\underbrace{\omega t + \frac{2\pi \sin\theta}{\lambda} a}_{A}\right) - \cos\left(\underbrace{\omega t}_{B}\right)\right]$$

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$(*) = \frac{\lambda E_0}{\pi a \sin\theta} \sin\left(\omega t + \frac{\pi \sin\theta}{\lambda} a\right) \sin\left(\frac{\pi \sin\theta}{\lambda} a\right)$$

$$\beta = \frac{2\pi \sin\theta}{\lambda} a$$

$$(*) = E_0 \frac{1}{(\beta/2)} \sin\left(\omega t + \frac{\beta}{2}\right) \sin\left(\frac{\beta}{2}\right)$$



$$(*) = E_0 \frac{1}{(\beta/2)} \sin\left(\omega t + \frac{\beta}{2}\right) \sin\left(\frac{\beta}{2}\right)$$

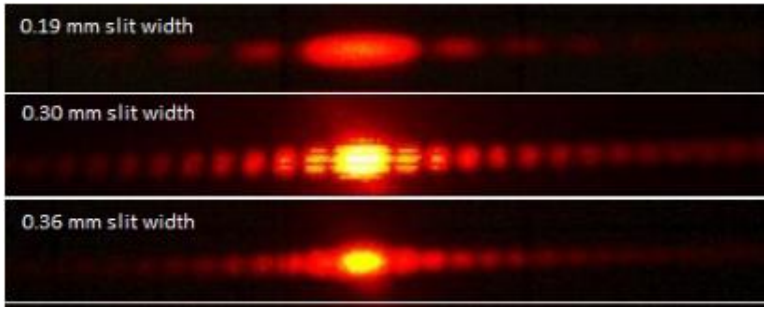
$$= E_\theta \sin(\omega t + \phi)$$

$$\Rightarrow E_\theta = E_0 \frac{1}{(\beta/2)} \sin(\beta/2)$$

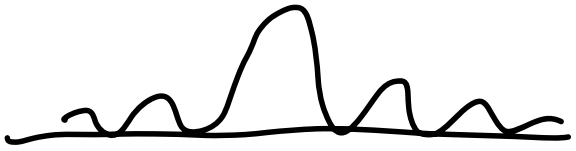
Intensity $\propto$ Amplitude^2

$$I(\theta) \propto E_0^2 \frac{\sin^2(\beta/2)}{(\beta/2)^2}$$

- Note that as wavelength λ increases, β decreases.
  - the bright spots spread-out.
- If the width a increases, β increases.
  - the bright spots will come closer together



$$I(\theta) \propto E_0^2 \frac{\sin^2(\beta/2)}{(\beta/2)^2} \qquad \beta = \frac{2\pi \sin\theta}{\lambda} a$$

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