$$\langle - | a_j a_i a_p^\dagger a_q a_\ell^\dagger a_k^\dagger | - \rangle$$

$$H = \sum_{pq=1}^{n} h_{pq} a_p^\dagger a_q \qquad [a_p, a_q^\dagger] = \delta_{pq}$$

$$\Downarrow$$

$$\langle ij | \sum_{pq} h_{pq} a_p^\dagger a_q | k\ell \rangle$$

$$\langle ij = (ij\rangle)^*$$

$$(a_j^\dagger a_i^\dagger)^* =$$

$$= (-1) \langle - | a_j a_p^\dagger a_i a_q a_\ell^\dagger a_k^\dagger | - \rangle$$

$$+ \delta_{pi} \langle - | a_j a_q a_\ell^\dagger a_k^\dagger | - \rangle$$

$$= \underbrace{(-1)^2 \langle - | a_p^\dagger a_j a_i a_q a_\ell^\dagger a_k^\dagger | - \rangle}_{0} \rightarrow 0$$

$$+ \delta_{pj} (-1) \langle - | a_i a_q a_\ell^\dagger a_k^\dagger | - \rangle$$

$$+ \delta_{pi} \delta_{q\ell} \langle - | a_j a_k^\dagger | - \rangle$$

$$+ (-1) \delta_{pi} \langle - | a_j a_\ell^\dagger a_q a_k^\dagger | - \rangle$$

$$a_p a_q^\dagger + \underline{a_q^\dagger a_p} = \delta_{pq}$$

$$a_p a_q^\dagger = \underline{\delta_{pq}} - a_q^\dagger a_p$$

$$+\ \delta_{pj}(-1)\langle\dashv a_i a_q a_l^\dagger a_k^\dagger \vdash\rangle \qquad (-1)\langle\dashv a_k^\dagger a_j \vdash\rangle \nearrow 0$$

$$+\ \delta_{pi}\delta_{ql}\langle\dashv a_j a_k^\dagger \vdash\rangle$$

$$+\ (-1)\delta_{pi}\langle\dashv a_j a_l^\dagger a_q a_k^\dagger \vdash\rangle$$

$$=\ \delta_{pj}\langle\dashv a_i a_l^\dagger a_q a_k^\dagger \vdash\rangle$$

$$+\ (-1)\delta_{pj}\delta_{ql}\langle\dashv a_i a_k^\dagger \vdash\rangle$$

$$+\ \delta_{pi}\delta_{ql}\delta_{jk}$$

$$+\ (-1)\delta_{pi}\delta_{jl}\langle\dashv a_q a_k^\dagger \vdash\rangle$$

$$\cancel{+}\ \delta_{pj}\delta_{il}\langle\dashv a_q a_k^\dagger \vdash\rangle \Rightarrow \delta_{pj}\delta_{il}\delta_{qk}$$

$$=\ -\delta_{pj}\delta_{ql}\delta_{ik}$$

$$+\ \delta_{pi}\delta_{ql}\delta_{jk}$$

$$-\ \delta_{pi}\delta_{jl}\delta_{qk}$$

$$+\ \delta_{pj}\delta_{il}\delta_{qk}$$

$$\langle ij| \sum_{p,q} h_{pq} \, a_p^\dagger a_q |kl\rangle \qquad p=j \quad q=l$$

$$= \sum_{pq} h_{pq} \Big(- \delta_{pj}\delta_{ql}\delta_{ik} + \delta_{pi}\delta_{ql}\delta_{jk} - \delta_{pi}\delta_{jl}\delta_{qk} + \delta_{pj}\delta_{il}\delta_{qk} \Big)$$

$$= -h_{j,l}\,\delta_{jk} + h_{i,l}\,\delta_{jk} - h_{i,k}\,\delta_{qk} + h_{j,k}\,\delta_{il}$$

$$O\left(\binom{n}{d}^3\right)$$

Wick's Theorem: $O(1)$

$$\langle \,|\, a_j a_i a_p^\dagger a_q a_l^\dagger a_k^\dagger \,|\, \rangle$$

$(-1)\,\delta_{ip}\,\delta_{jl}\,\delta_{qk}$ $\qquad O(1)$

$(-1)^0\,\delta_{jk}\,\delta_{ip}\,\delta_{ql}$

$(-1)^2\,\delta_{jp}\,\delta_{il}\,\delta_{qk}$

$(-1)\,\delta_{jp}\,\delta_{ik}\,\delta_{ql}$

$$\Big(\; -\,\delta_{pj}\,\delta_{ql}\,\delta_{ik} + \delta_{pi}\,\delta_{ql}\,\delta_{jk} - \delta_{pi}\,\delta_{jl}\,\delta_{qk} + \delta_{pj}\,\delta_{il}\,\delta_{qk}$$

Slater-Condon Rules
- Wikipedia